Models and examples in linear optimization

Example 1. In order to finance the development of a combine through the next 5 years the managing council has presented funds (in thousands of dollars) for each year as follows:

year	2008	2009	2010	2011	2012
funds	3500	4500	4000	3450	4500

Suggestions for the funding of 15 projects have been received with the needed funds for each year and expected profits for each project are listed below:

Project	2008	2009	2010	2011	2012	Profit
1	400	230	330	400	230	1500
2	500	270	260	500	270	2000
3	200	430	430	200	430	2500
4	300	220	270	300	220	7000
5	450	500	400	450	500	4000
6	650	450	320	650	450	3000
7	350	530	330	350	530	4500
8	300	700	260	300	700	3500
9	500	800	430	500	800	1500
10	600	250	270	600	250	2000
11	150	500	400	150	500	2500
12	250	340	320	250	340	7000
13	220	400	250	220	270	4000
14	170	300	300	170	300	3000
15	420	400	260	420	400	4500

It must be decided which projects to finance in order to receive the best possible profit by taking into account the available funds for each year.

Solution:

Let the variables x_j assume the value of 0 if the jth project is rejected and 1 if it is funded (j=1,2,….,15).

Then the expected profit will be $1500X_1+2000X_2+…+4500X_{15}$

The early expense will be:

2008: $400X_1+500X_2+…+420X_{15}$

2009: $230X_1+270X_2+…+400X_{15}$

2010: $330X_1+260X_2+…+260X_{15}$

2011: $400X_1+500X_2+…+420X_{15}$

2012: $230X_1+270X_2+…+400X_{15}$

That way we get the following linear optimization problem:

Max $1500X_1+2000X_2+…+4500X_{15}$

With constraints:

$$400X_1+500X_2+…+420X_{15} \leq 3500$$
$$230X_1+270X_2+…+400X_{15} \leq 4500$$
$$330X_1+260X_2+…+260X_{15} \leq 4000$$
$$400X_1+500X_2+…+420X_{15} \leq 3450$$
$$230X_1+270X_2+…+400X_{15} \leq 4050$$
$$X_j = \{0,1\} \text{ за } j=1,2,…,15$$

Answer

The answer to the problem is as follows: only projects numbered 3, 4, 5, 6, 7, 8, 12, 13, 14 and 15 are financed. The expected profit is 43000 Thousand dollars and the expected expenses are:

Year	2008	2009	2010	2011	2012
Expense	3310	4270	3140	3310	4140

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